

Exact Master Equation and Non-Markovian Decoherence for Quantum Dot Quantum Computing

M. W. Y. Tu · M. T. Lee · W. M. Zhang*

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Abstract In this article, we report the recent progress on decoherence dynamics of electrons in quantum dot quantum computing systems using the exact master equation we derived recently based on the Feynman-Vernon influence functional approach. The exact master equation is valid for general nanostructure systems coupled to multi-reservoirs with arbitrary spectral densities, temperatures and biases. We take the double quantum dot charge qubit system as a specific example, and discuss in details the decoherence dynamics of the charge qubit under coherence controls. The decoherence dynamics risen from the entanglement between the system and the environment is mainly non-Markovian. We further discuss the decoherence of the double-dot charge qubit induced by quantum point contact (QPC) measurement where the master equation is re-derived using the Keldysh non-equilibrium Green function technique due to the non-linear coupling between the charge qubit and the QPC. The non-Markovian decoherence dynamics in the measurement processes is extensively discussed as well.

Keywords quantum decoherence, nanoelectronic devices, quantum computation, functional analytic methods, non-equilibrium dynamics, quantum measurements.

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Matisse Wei-Yuan Tu

Department of Physics and Center for Quantum Information Science, National Cheng Kung University, Tainan, 70101 Taiwan.

Present address: Institute for Materials Science and Max Bergmann Center of Biomaterials, Dresden University of Technology, D-01069 Dresden, Germany.

Ming-Tsung Lee

National Center for Theoretical Science, Tainan, 70101 Taiwan.

Present address: Research Center for Applied Science, Academic Sinica, Taipei, 11529 Taiwan.

Wei-Min Zhang

Department of Physics and Center for Quantum Information Science, National Cheng Kung University, Tainan, 70101 Taiwan; National Center for Theoretical Science, Tainan, 70101 Taiwan.

*E-mail: wzhang@mail.ncku.edu.tw

1 Introduction

The investigation of quantum coherence and quantum entanglement in quantum dot systems has been attracting much attention in the past decade because of its scalability for quantum computer. Due to the rapid development of nanotechnology, implementing quantum information processing using nanostructures is very prospective. Among a diversity of nanostructures, a prototypical one is made by fabrications on the interface of a heterostructure semiconductor as a gate-defined region containing a set of coupled electronic states. Electrodes are implanted around this region to control the bias across it, and also several gates are implemented to adjust the electronic states within the central area as well as their couplings to the surrounding electrodes. Because the electronic structures of these systems can be well controlled, they have been intensively and extensively studied for many purposes including understanding fundamental physical issues and building useful quantum devices. However, it is also because of its openness to the controlling electrodes, the external gates and various degrees of freedom in the host material, the central electronic region is intimately entangled with the surrounding environment. Quantum entanglement among the basic units for quantum information processing, i.e. quantum bits or simply qubits, is the unique resource for exponentially speedup over classical algorithms. However, quantum entanglement between the qubit system and its environment is indeed a debt to quantum information processing. The entanglement between the system and its environment induces severe loss of quantum coherence of qubits and destroys eventually the useful entanglement among qubits. It has been realized that decoherence is the most difficult obstacle for any successful quantum information processing. Therefore, the investigation of decoherence dynamics due to the entanglement of the qubit systems with the surrounding environment and the development of decoherence control protocols become the central issue in the study of quantum information processing. In this article, we will present a general theory for the description of the real time dynamics of a central electronic system coupled to multi-electrodes. The theory is build on the exact master equation for general nanostructures we derived recently using the Feynman-Vernon influence functional approach [1]. This exact master equation is capable to depict the detailed decoherence dynamics of electronic states in the nanostructure with arbitrary spectral density at an arbitrary initial temperature of the reservoirs and an arbitrary bias applied to the reservoirs.

Historically, since it was first proposed by Feynman and Vernon in 1963 [2] for quantum Brownian motion (QBM) modelled as a central harmonic oscillator linearly coupled to a set of harmonic oscillators simulating the thermal bath, the influence functional approach has been widely used to study dissipation dynamics in quantum tunneling problems [3] and decoherence problems in quantum measurement theory [4,5]. In the early applications, the master equation was derived for some particular class of Ohmic (white-noise) environment [6]. The exact master equation for the QBM with a general spectral density (color-noise environments) at arbitrary temperature was obtained by Hu and co-workers in 1992 [7]. Applications of the QBM exact master equation cover various topics, such as quantum decoherence, quantum-to-classical transition, quantum measurement theory, and quantum gravity and quantum cosmology, etc. [8,9,10]. Very recently, such an exact master equation is further extended to the systems of two entangled harmonic oscillators [11] and two entangled optical fields [12,13] for the study of non-Markovian entanglement dynamics. Nevertheless, using the influence functional approach to obtain the exact master equation has been largely focused on the bosonic environments in the past half century. The extension of the

influence functional approach to fermion environments is just started [1], where the Feynman path integral in terms of fermion coherent states [14] must be used.

In fact, the investigation of the environmental effects on the subsequent dynamics of the principal system one concerned is one of the essential issues in many currently interesting research topics related to quantum decoherence and dissipation phenomena. The quantum decoherence and dissipation phenomena are also associated with quantum non-equilibrium dynamics, a subject that has been investigated over a half century yet is still not completely understood. Quantum decoherence in quantum computations and quantum information processes is one of the biggest challenges for practical applications of quantum mechanics. Using the Feynman-Vernon influence functional approach to understand decoherence phenomena is fundamental in the sense that it allows to completely integrate out the environmental degrees of freedom. As a result, the back-action of the environment to the system can be fully taken into account. The resulting non-perturbation (with respect to the coupling between the system and its environment) master equation is indeed desirable for quantum information processing since the fast manipulations to qubit states requires a strong coupling among the constituents. It enables us to precisely explore the real-time dynamics of the electron charge coherence in various different nanostructures under different manipulating conditions.

On the other hand, quantum measurement (readout of qubit states) also often induces severe decoherence. Therefore, precision readout of qubit states is another big challenge in quantum information processing. Again due to the rapid progress in nanotechnology, electron state readout in nanostructures has been investigated experimentally with qubit systems coupling to a mesoscopic measurement device, such as single electron transistor (SET) or quantum point contact (QPC). Not only its practical application to quantum information processing, but also the theoretical interests in the measurement-induced quantum decoherence have attracted much attention. In particular, in the investigation of quantum dot quantum computing with charge qubits, the QPC has been served as an ultrasensitive electrometer. The theoretical understanding of charge qubit measurement through the QPC was previously treated based on the Markov approximation [15,16,17] in which the time scale of the qubit dynamics is assumed much longer than that of the tunneling-electron correlation in the QPC. The treatments based on the Markov approximation, in fact, only describe the qubit dynamics in the time-asymptotic quasi-equilibrium state. However, for the quantum measurement in terms of SET or QPC, the qubit decoherence occurs in the time scale of the same order of the tunneling-electron correlation time in the measurement equipments, where the non-Markovian dynamics of the qubit is significant. In the second part of this article we shall discuss a fully non-equilibrium theory we have developed recently [18,19] for understanding the qubit decoherence induced by the QPC measurement, based on the Keldysh non-equilibrium Green function technique [20].

The Keldysh non-equilibrium Green function approach is well developed for a perturbation treatment to the non-equilibrium dynamics in many-body systems [21,22]. This approach has been used to study various transport phenomena in nanostructures [23]. To make the non-equilibrium effect of the electrical reservoir more transparent, a real-time diagrammatic technique was constructed to diagrammatically calculate correlation functions of the electrical reservoir order by order in the perturbation expansion [24,25]. In the investigation of the non-Markovian dynamics in the double dot charge qubit induced by the QPC measurement, we developed an alternative real-time diagrammatic technique [18]. The master equation for the charge qubit dynamics is derived and expressed in terms of irreducible diagrams up to all orders. The effect of

the fluctuant measurement reservoir due to the interaction with the qubit system can be fully taken into account. The measurement-induced non-Markovian decoherence in the qubit dynamics can be explicitly studied in this formulism.

2 Exact master equation of electron transport systems

The prototypical nanostructure under consideration can be generally described by the following electronic Hamiltonian

$$H = \sum_{ij} \epsilon_{ij} d_i^\dagger d_j + \sum_{ij} U_{ij} d_i^\dagger d_i d_j^\dagger d_j + \sum_{\alpha k} \epsilon_{\alpha k} c_{\alpha k}^\dagger c_{\alpha k} + \sum_{i\alpha k} (t_{i\alpha k} d_i^\dagger c_{\alpha k} + t_{i\alpha k}^* c_{\alpha k}^\dagger d_i), \quad (1)$$

where the first two terms are the general Hamiltonian H_D of electrons in the central region of the nanostructure. For simplicity, we shall ignore the electron-electron interaction in this article, i.e., let $U_{ij} = 0$. The third term is the Hamiltonian H_R describing the noninteracting electron reservoirs (the source and drain electrodes, etc.) labeled by the index α . The last term is the electron tunneling Hamiltonian H_T between the reservoirs and the central system. $d_i^\dagger (d_i)$ and $c_{\alpha k}^\dagger (c_{\alpha k})$ are the electron creation (annihilation) operators of the central system and the surrounding reservoirs, respectively. Also, throughout this work, we set $\hbar = 1$. The initial inverse temperature for the α -lead is denoted as $\beta_\alpha = 1/k_B T_\alpha$.

Using the Feynman-Vernon influence functional approach extended to fermion coherent state representation [14], we can integrate out completely the degrees of freedom of the electron reservoirs and obtain the following exact master equation [1],

$$\begin{aligned} \dot{\rho}(t) = & -i[H_D'(t), \rho(t)] \\ & + \sum_{ij} \{ \Gamma_{ij}(t) [2d_j \rho(t) d_i^\dagger - d_i^\dagger d_j \rho(t) - \rho(t) d_i^\dagger d_j] \\ & + \Gamma_{ij}^\beta(t) [d_j \rho(t) d_i^\dagger - d_i^\dagger \rho(t) d_j - d_i^\dagger d_j \rho(t) + \rho(t) d_j d_i^\dagger] \}. \end{aligned} \quad (2)$$

This master equation determines completely the quantum coherence dynamics of the central electronic system. $\rho(t)$ is the reduced density matrix of the central system at time t after the environmental degrees of freedom are completely eliminated. $H_D'(t) = \sum_{ij} \epsilon'_{ij}(t) d_i^\dagger d_j$ is the corresponding renormalized Hamiltonian. Other two non-unitary terms in the master equation describe the reservoir-induced dissipative and noise processes with time-dependent dissipation and fluctuation coefficients, $\Gamma(t)$ and $\Gamma^\beta(t)$. All these time-dependent coefficients in the master equation are derived non-perturbatively and are exact. The time-dependent fluctuations of the energy levels, $\epsilon'_{ii}(t)$, and the transition couplings between states, $\epsilon'_{ij}(t)$ for $i \neq j$, are the renormalization effects that contain the energy level shifts and the coupling changes between them, while $\Gamma(t)$ and $\Gamma^\beta(t)$ are the dissipation and fluctuation effects that depict the full non-Markovian decoherence dynamics. These are all the back-action effects risen from the electron tunneling processes between the central system and the reservoirs.

The explicit time dependencies of these coefficients in the master equation are given as follows

$$\epsilon'_{ij}(t) = \frac{i}{2} [\dot{\mathbf{u}}(t) \mathbf{u}^{-1}(t) - (\mathbf{u}^\dagger)^{-1}(t) \dot{\mathbf{u}}^\dagger(t)]_{ij}, \quad (3a)$$

$$\Gamma_{ij}(t) = -\frac{1}{2} [\dot{\mathbf{u}}(t) \mathbf{u}^{-1}(t) + (\mathbf{u}^\dagger)^{-1}(t) \dot{\mathbf{u}}^\dagger(t)]_{ij}, \quad (3b)$$

$$\Gamma_{ij}^\beta(t) = [\dot{\mathbf{u}}(t) \mathbf{u}^{-1}(t) \mathbf{v}(t) + \mathbf{v}(t) (\mathbf{u}^\dagger)^{-1}(t) \dot{\mathbf{u}}^\dagger(t) - \dot{\mathbf{v}}(t)]_{ij}. \quad (3c)$$

The elementary functions $\mathbf{u}(t)$ and $\mathbf{v}(t)$ are time-dependent matrices following the dissipation-fluctuation integral-differential equations

$$\dot{\mathbf{u}}(\tau) + i\epsilon \mathbf{u}(\tau) + \sum_{\alpha} \int_{t_0}^{\tau} d\tau' \mathbf{F}_{\alpha}(\tau - \tau') \mathbf{u}(\tau') = 0, \quad (4a)$$

$$\dot{\mathbf{v}}(\tau) + i\epsilon \mathbf{v}(\tau) + \sum_{\alpha} \int_{t_0}^{\tau} d\tau' \mathbf{F}_{\alpha}(\tau - \tau') \mathbf{v}(\tau') = \sum_{\alpha} \int_{t_0}^t d\tau' \mathbf{F}_{\alpha}^{\beta}(\tau - \tau') \bar{\mathbf{u}}(\tau'), \quad (4b)$$

subjected to the boundary conditions $\mathbf{u}(t_0) = \mathbf{I}$ and $\mathbf{v}(t_0) = 0$, where $\bar{\mathbf{u}}(\tau) = \mathbf{u}^\dagger(t - \tau + t_0)$. Introduce the spectral density $J_{ij}^{\alpha}(\epsilon) = 2\pi \sum_{\mathbf{k} \in \alpha} t_{i\alpha\mathbf{k}}^* t_{j\alpha\mathbf{k}} \delta(\epsilon - \epsilon_{\alpha\mathbf{k}})$, then the integration kernels in the above equations can be expressed as $F_{ij}(\tau - \tau') = \sum_{\alpha} \int \frac{d\epsilon}{2\pi} J_{ij}^{\alpha}(\epsilon) e^{-i\epsilon(\tau - \tau')}$, and $F_{ij}^{\beta}(\tau - \tau') = \sum_{\alpha} \int \frac{d\epsilon}{2\pi} f_{\alpha}(\epsilon) J_{ij}^{\alpha}(\epsilon) e^{-i\epsilon(\tau - \tau')}$, where $f_{\alpha}(\epsilon) = \frac{1}{e^{\beta_{\alpha}(\epsilon - \mu_{\alpha})} + 1}$ is the initial fermi distribution function of the reservoir α held at the chemical potential μ_{α} . These are the so-called dissipation-fluctuation kernels and represent dissipation and fluctuation effects on the central system due to its coupling to electron reservoirs. The relation between these two kernels are then seen from the dissipation-fluctuation theorem. The real time non-equilibrium electron dynamics of the central systems is fully described by the master equation.

3 Coherence control of double dot charge qubit

In the quantum computing scheme in terms of double quantum dots where the electron charge degree of freedom is exploited, the effects in deviating the coherency of charge dynamics are usually summarized in the fluctuations of the inter-dot coupling and energy splitting between the two local charge states. The amplitudes of these fluctuations may be estimated from measurements of the noise spectrum of electron currents and the minimum line width of elastic current peak [26]. Parallel theoretical works have been developed with different approaches in the literature for the purposes of both simulating the experimental results and understanding the physical mechanisms living in the double-dot. With the master equation shown above, a complete investigation can be carried out for the decoherence dynamics of the double dot charge qubit under coherence controls. The decoherence of the electron charges can be addressed with the back-reaction effects of the reservoirs to the double dot (including not only the fluctuations of the inter-dot coupling and energy splitting between the two local charge states but also the non-unitary dynamical effect induced by dissipations and noises) being fully taken into account.

For the double dot charge qubit (a schematic plot see Fig. 1), the corresponding Hamiltonian of the central system becomes

$$H_{\text{dot}} = E_1 d_1^{\dagger} d_1 + E_2 d_2^{\dagger} d_2 + \Omega_0 (d_2^{\dagger} d_1 + d_1^{\dagger} d_2), \quad (5)$$

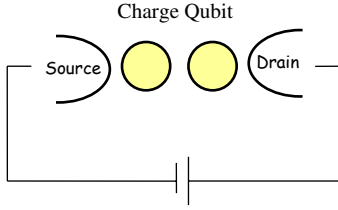


Fig. 1 (color online). A schematic plot of the double quantum dot coupled to the source and drain electrodes.

where E_1 and E_2 are the on-site energies of the left and the right dot respectively, Ω_0 is the tunnel coupling between the two sites. The double quantum dot is coupled to two electron leads on the left and the right side. Specifically, the left dot is only coupled to the left lead while the right dot is only coupled to the right lead. To closely monitor the time evolution of the electronic states in this system, it is more convenient to specify the master equation in the particle number basis of the no electron state $|0\rangle$, the state of one electron in the left dot $|1\rangle$, one electron in the right dot $|2\rangle$, and the double occupied state of two electrons in two dots $|3\rangle$. Then the master equation is reduced to a set of coupled rate equations in this basis. Furthermore, the double dot charge qubit works in the single-electron regime, where the strong Coulomb blockade (Coulomb repulsion between the two sites) plays an important role. In other words, the doubly occupied state $|3\rangle$ must be excluded physically. This can be easily taken into account by excluding the double occupancy from the rate equations properly. The consequent rate equations excluding the double occupancy read [1]

$$\dot{\rho}_{00} = \tilde{\Gamma}_{11}\rho_{11} + \tilde{\Gamma}_{21}\rho_{12} + \tilde{\Gamma}_{12}\rho_{21} + \tilde{\Gamma}_{22}\rho_{22} + \text{Tr}\Gamma^\beta\rho_{00}, \quad (6a)$$

$$\dot{\rho}_{11} = -\tilde{\Gamma}_{11}\rho_{11} + \tilde{\Xi}_-^*\rho_{12} + \tilde{\Xi}_-\rho_{21} - \Gamma_{11}^\beta\rho_{00}, \quad (6b)$$

$$\dot{\rho}_{22} = -\tilde{\Gamma}_{22}\rho_{22} + \tilde{\Xi}_+^*\rho_{12} + \tilde{\Xi}_+\rho_{21} - \Gamma_{22}^\beta\rho_{00}, \quad (6c)$$

$$\dot{\rho}_{12} = [-i\epsilon' - \frac{1}{2}\text{tr}\tilde{\Gamma}]\rho_{12} + \tilde{\Xi}_+\rho_{11} + \tilde{\Xi}_-\rho_{22} - \Gamma_{12}^\beta\rho_{00}, \quad (6d)$$

where $\tilde{\Gamma}(t) = 2\Gamma(t) + \Gamma^\beta(t)$, $\tilde{\Xi}_\pm(t) = \pm i\epsilon'_{12}(t) - \frac{1}{2}\tilde{\Gamma}_{12}(t)$ and $\epsilon'(t) = \epsilon'_{11}(t) - \epsilon'_{22}(t)$, $\Gamma(t)$ and $\Gamma^\beta(t)$ are the time-dependent coefficient matrices in the master equation (2) with $i = 1, 2$. We can reproduce other rate equations from (6) in the corresponding Born-Markov limit used in the literature [27, 28, 29].

To study the decoherence dynamics of the charge qubit, the spectral density must be specified. Here we use a Lorentzian spectral density. For the double dot charge qubit system, since the left (right) dot only couples to the left (right) electrode, the spectral density can be written explicitly as

$$J_{ij}^\alpha(\epsilon) = \frac{\Gamma_\alpha W_\alpha^2}{(\epsilon - E_i)^2 + W_\alpha^2} \delta_{ij}, \quad (7)$$

with $\alpha = L(R)$ for $i = 1(2)$. The Lorentzian spectral widths $W_{L,R}$ are indeed the bandwidths of the densities of states for the source and drain, respectively. They depict the characteristic times of the corresponding reservoirs. The larger the widths $W_{L,R}$ are, the shorter the correlation time of the reservoirs give and the lesser the memory effect shows. In the wide band limit (WBL) that is widely used in the literature, the

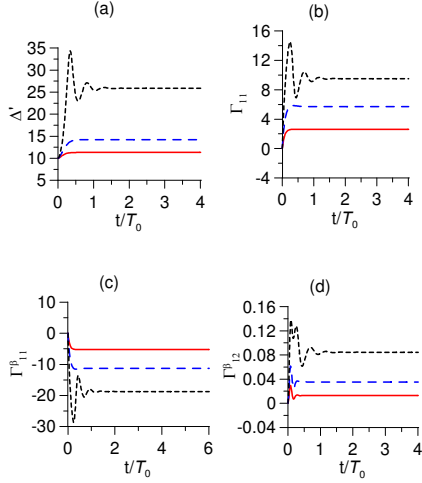


Fig. 2 (color online). The time dependence of transport coefficients in the master equation where $\Delta'(t) = 2\epsilon'_{12}(t)$. The inter-dot tunnel coupling Ω_0 is set to be $5\mu\text{eV}$ with the fixed chemical potentials at $\mu - E = 50\mu\text{eV}$ and the Lorentzian spectral width $W = 20\mu\text{eV}$. The red line, the blue dashed line and the black dot lines correspond to $\Gamma = 5, 10$ and $25\mu\text{eV}$, respectively.

spectral density becomes energy independent. As a matter of fact, all the memory effect is wiped out. The coupling constants $\Gamma_{L,R}$ are the tunneling rates of electron tunneled into the double dot from the source and the drain, respectively. The tunneling rates $\Gamma_{L,R}$ describe the strength of the tunneling process between the dot and the reservoirs covering the leakage effect, and are experimentally adjustable through gate voltages. Varying the coupling strength to the environment and the correlation time scale of the reservoirs, we are able to analyze realistically the decoherence dynamics of the double dot charge qubit.

For manipulating the charge qubit coherence, the device is set at the resonant condition $E_1 = E_2 = E$. The fermi surfaces of the two electrodes are aligned $\mu_L = \mu_R = \mu$ above the resonant level $\mu - E > 0$ so that electrons are kept in the dot with minor probability of leakage. To simplify the problem, we consider only the symmetric double dot system where $\Gamma_L = \Gamma_R = \Gamma$ and $W_L = W_R = W$. The electron temperature is kept at 100mK throughout the rest of the calculations. The time-dependence of the transport coefficients in the master equation is essential for the non-Markovian dynamics during the coherence controls of charge qubit states. Here we first examine the time dependent behavior of these transport coefficients. Fig. 2 shows an example of the time dependencies of those coefficients at a few different strengths of coupling (tunneling rate) between the qubit and electron reservoirs, where the coupling strength can be controlled by tuning the gate voltages in experiments. The time is in unit of $T_0 = 2\pi/\sqrt{(E_1 - E_2)^2 + 4\Omega_0^2} = \pi/\Omega_0$. As one can see the stronger the coupling between the qubit and the environment is, the larger fluctuation the coefficients shows that reveals the non-Markov feature. As it is known the rate between the bandwidth W and the tunneling rate Γ tells how important the non-Markovian dynamics can be. The width W characterizes the correlation time of the electron reservoirs, a large width W

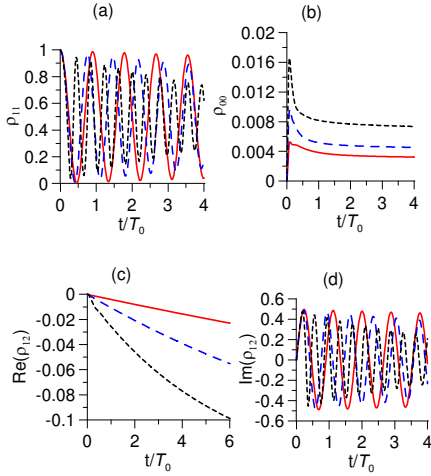


Fig. 3 (color online). The time evolution of the reduced density matrix elements for the charge qubit solved from the master equation. All the parameters are the same as that given in Fig. 2

corresponds to a short characteristic time of the reservoir. Therefore, a smaller W/Γ will make the non-Markovian process more obvious in electron dynamics. This feature is shown by the time-dependence of the transport coefficients, as we can see from Fig. 2. When W/Γ becomes large enough, the time-dependence of the transport coefficients will disappear and the Markovian limit may be reached. Equivalently speaking, a larger width W corresponds to a shorter correlation time of the electron reservoir, the coefficients at larger width reach their steady limit sooner.

Correspondingly, the dynamics of the charge qubit is also examined at different coupling strengths by plotting the time evolution of the reduced density matrix elements. Fig. 3(a)-(d) show the time evolution of the electron population occupying the first dot given by $\rho_{11}(t)$, the probability of empty dots, i.e. the leakage effect by $\rho_{00}(t)$, and the real and imaginary parts of the off-diagonal matrix element $\rho_{12}(t)$. For the symmetric dots ($E_1 = E_2$) concerned here, the initial energy eigenstates of the double dots are given by the bonding and anti-bonding states, $|\pm\rangle = \frac{1}{\sqrt{2}}(|1\rangle \pm |2\rangle)$. The relaxation and decoherence of the qubit states are then determined by the decays of the matrix elements $\langle +|\rho|+ \rangle = \frac{1}{2}(\rho_{11} + \rho_{22}) + \text{Re}\rho_{12}$ and $\langle +|\rho|- \rangle = \frac{1}{2}(\rho_{11} - \rho_{22}) - i\text{Im}\rho_{12}$, respectively. In other words, the decay rate of $\text{Re}\rho_{12}(t)$ is the relaxation time T_1 (because $\rho_{11} + \rho_{22} \simeq 1$) while the decay rates of $\rho_{11}(t), \rho_{22}(t)$ or $\text{Im}\rho_{12}(t)$ give the decoherence time T_2 , as we have discussed in [1]. These properties are clearly shown in Fig. 3, where the oscillation decays in Fig. 3(a) and (d) depict the decoherence dynamics while Fig. 3(c) describes the relaxation process (energy dissipation) and Fig. 3(b) shows the small leakage effect. We find that the decay of the charge coherent oscillation is well described by a simple exponential decay for the off-diagonal reduced density matrix elements. The diagonal matrix elements (populations) are better described by a sub-exponential law when charge leakage is not negligible, otherwise simple exponential decay is better for all the cases. The relaxation time T_1 and the decoherence time T_2 can be extracted from the exact numerical solution $\text{Re}\rho_{12}(t)$ and $\text{Im}\rho_{12}(t)$, respectively,

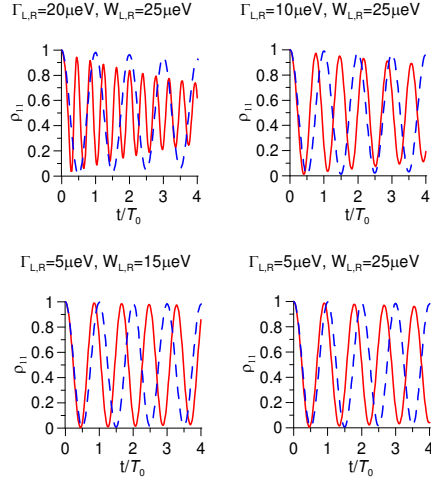


Fig. 4 (color online). A comparison of the exact solution (red lines) with the Markov limit (blue dashed lines), where the inter-dot tunnel coupling $\Omega_0 = 5\mu\text{eV}$ with the fixed chemical potential at $\mu - E = 50\mu\text{eV}$.

with the result $T_1 \leq 2T_2$ being of the order of a few nanoseconds or less for a broad parameter range we used [1].

Furthermore, a stronger tunneling coupling Γ to the electron reservoirs (compare to the Lorentzian spectral width W) leads to a faster decay of the coherent charge qubit oscillation and also causes severer shifts in the charge oscillation frequency. The non-Markovian decoherence dynamics of charge qubit is dominated by two major effects, the memory effect and the leakage effect in the double-dot gated by electrode reservoirs. The former becomes a dominate effect when the time scale (the inverse of the width W) of the reservoirs is comparable to the time scale ($\sim T_0$) of the double-dot. The latter becomes important when the electron tunneling strength between the reservoirs and dots is tuned to be large. Strengthening the couplings between the reservoirs and dots disturbs the charge coherence in the double-dot significantly. However, reasonably raising up the chemical potentials $\mu_{L,R}$ can suppress charge leakage and maintain charge coherence. The left uncontrollable decoherence factor is the spectral width which characters how many electron states in the reservoirs effectively involving in the tunneling processes between the dots and the reservoirs. It is quite interest to see that in any case, the Markov limit (based on the rate equations derived by Brandes and Vorrath under Born-Markov approximation but without including the phonon coupling [29]) deviates largely from the exact solutions, as shown in Fig. 4. It indicates that quantum information processing is indeed non-Markovian with the double dot charge qubit.

4 Charge qubit with QPC measurement

In this section, we present a study of the charge qubit measurement using QPC in the tunnel junction regime. The transmissions of all tunneling channels cross the QPC barrier are required to be small enough in this regime such that the electron tunneling

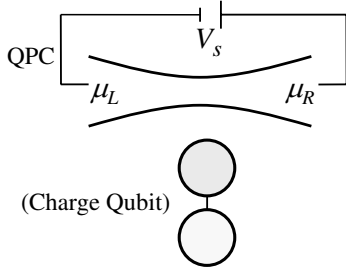


Fig. 5 A schematic plot for the QPC measurement of double dot charge qubit.

becomes sensitive to the qubit state. The qubit information can then be extracted from the output signal of the QPC, while the back-action of the measurement to the qubit states can also be taken into account. In order to explore the qubit decoherence induced by the QPC measurement, the corresponding Hamiltonian takes the form [15]

$$H = H_{\text{dot}} + \sum_{\alpha, k} \epsilon_{\alpha k} a_{\alpha k}^{\dagger} a_{\alpha k} + \sum_{kk'} (q_{kk'} a_{Rk'}^{\dagger} a_{Lk} + q_{kk'}^{\dagger} a_{Lk}^{\dagger} a_{Rk'}). \quad (8)$$

where H_{dot} is the Hamiltonian of the double dot charge qubit given by Eq. (5). The second term is the Hamiltonian of the QPC electronic reservoirs consisting of the source ($\alpha = L$) and the drain ($\alpha = R$) electrodes with the energy levels $\epsilon_{\alpha k}$, and the chemical potentials μ_L and μ_R , respectively. The last term is the interaction Hamiltonian describing the electron tunneling processes through the QPC with a qubit-state dependent hopping amplitude $q_{kk'} = \Omega_{kk'} - \delta\Omega_{kk'} d_1^{\dagger} d_1$. The single electron occupies on the first dot leads to a variation in the barrier of the QPC, and the electron hopping amplitude is thus modified from $\Omega_{kk'}$ to $\Omega'_{kk'} = \Omega_{kk'} - \delta\Omega_{kk'}$. The charge qubit state is measured through the electron tunneling across the source and the drain. A schematic device is plotted in Fig. 5.

To explore the measurement-induced decoherence of charge qubit, the QPC can be considered as an environment coupled to the charge qubit. Then the qubit dynamics is determined by the master equation for the reduced density operator $\rho(t) = \text{tr}_R[\rho_{\text{tot}}(t)]$, where $\rho_{\text{tot}}(t)$ is the total density operator for the whole system of the qubit plus QPC reservoir, and the partial trace tr_R integrates over all the degrees of freedom of the QPC reservoir. Since the qubit-reservoir coupling involves two-body interactions, the influence functional approach for integrating over all the degrees of freedom of the QPC reservoir cannot be exactly carried out. Therefore, we developed a real-time diagrammatic technique based on the Keldysh non-equilibrium Green function approach to study the decoherence effect of non-equilibrium electrodes on the qubit. The master equation for the charge qubit coupled with the QPC measurement has been derived in terms of the irreducible diagrams to all the orders in the perturbation expansion [18, 19],

$$\dot{\rho}(t) = -i[H_S, \rho(t)] - \sum_{n=0}^{\infty} \int_{t_0}^t d\tau K_{\text{ir}}^{(2n)}(t - \tau) * \rho(\tau), \quad (9)$$

where the kernel expansion $K_{\text{ir}}^{(2n)}(t - \tau) * \rho(\tau)$ consists of all the irreducible diagrams containing $2n + 2$ -particle correlations. The irreducible diagrams are defined by all

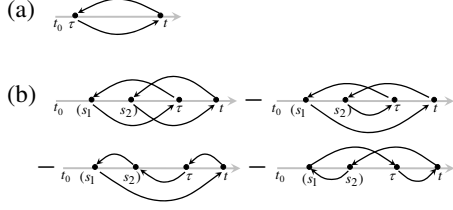


Fig. 6 Irreducible diagrams: (a) and (b) are the leading $K_{\text{ir}}^{(0)}(t - \tau)$ and the second leading-order $K_{\text{ir}}^{(2)}(t - \tau)$ contributions, respectively.

connected topology-independent diagrams along the real time axis. The connected diagram means that each loop in the diagram should intersect with other loops at least once. Physically, in the tunneling processes classified by $K_{\text{ir}}^{(2n)}(t - \tau)$, there must exist at least two particles correlating together in arbitrary time interval of the time period from t_0 to t . The leading and the second-order irreducible diagrams are shown in Fig. 6, where each vertex depends on the qubit operator via $q_{kk'}$ and the connecting free propagator specifies the non-equilibrium effect of the tunneling electron QPC on the qubit through the Keldysh Green function matrix. For the detailed diagrammatic rules and the classification of topology-independent irreducible diagrams, please refer to [18,19].

Since the QPC is often used for continuous measurement of charge qubit states, the coupling between the qubit and the QPC can be treated as a weak coupling. For the study of this QPC measurement-induced non-Markovian decoherence to the qubit states, it may be sufficient to consider only the leading order contribution to the master equation. The master equation (up to the leading order) is given as follows

$$\dot{\rho}(t) = -i[H_S, \rho(t)] - \int_{t_0}^t d\tau [R_0, [[K(t - \tau), \tilde{\rho}(\tau)]]], \quad (10)$$

where $\tilde{\rho}(\tau) \equiv e^{-iH_S(t-\tau)}\rho(\tau)e^{iH_S(t-\tau)}$, the double bracket $[[A, B]] \equiv AB - (AB)^+$, the operator $R_0 = R(0)$ and $R(t)$ is defined as $R(t) = \cos \theta(|e\rangle\langle e| - |g\rangle\langle g|) - \sin \theta(e^{i\gamma t}|g\rangle\langle e| + e^{-i\gamma t}|e\rangle\langle g|)$ with $\gamma = \sqrt{4\Omega_0^2 + (E_1 - E_2)^2}$ the energy difference between the ground state $|g\rangle$ and the excited state $|e\rangle$ of the qubit and $\theta = \cos^{-1}[(E_1 - E_2)/\gamma]$. The operator $K(t) = k(t)R(t)$, and $k(t)$ the reservoir correlation function that characterizes the QPC structure. It can be expressed explicitly as

$$k(t) = \int \frac{d\epsilon d\epsilon'}{(2\pi)^2} e^{i(\epsilon - \epsilon')t} \{f_L(\epsilon)[1 - f_R(\epsilon')]J(\epsilon, \epsilon') + f_R(\epsilon)[1 - f_L(\epsilon')]J(\epsilon', \epsilon)\}, \quad (11)$$

where $f_{L,R}(\epsilon) = 1/(1 + \exp \beta(\epsilon - \mu_{L,R}))$ are the Fermi-Dirac distribution functions for the source and the drain, $J(\epsilon, \epsilon')$ is the spectral density of the QPC reservoir which is defined by

$$J(\epsilon, \epsilon') = \pi^2 g_L(\epsilon) g_R(\epsilon') |\delta\Omega(\epsilon, \epsilon')|^2, \quad (12)$$

with $g_{L,R}(\epsilon)$ being the density of states of the source and the drain, and $\delta\Omega(\epsilon, \epsilon')$ the difference of the energy-dependent hopping amplitude (between the source and the drain) without and with the occupation of the first dot.

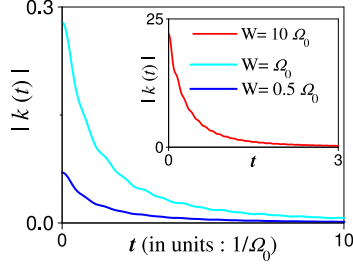


Fig. 7 (color online). The amplitude of the reservoir time correlation function during the measurement with different width W . Here we take the parameters $w = 2\Omega_0$, $\beta = 1/\Omega_0$, $V_d = 20\Omega_0$ and $\Gamma_d = \Omega_0^2/4$, respectively. The amplitude is plotted in units Ω_0^2 .

A close connection between the qubit decoherence and the tunneling-electron fluctuation is revealed through the reservoir time correlation function $k(t - \tau)$ of Eq. (11). The reservoir time correlation function describes the variation of the tunneling-electron correlation during the measurement. As we mentioned in the introduction, the previous investigations for the QPC measurement-induced decoherence of charge qubit are mainly focused on the Markov limit [15, 16, 17] in which the time scale of the qubit dynamics is assumed much longer than that of the tunneling-electron correlation in the QPC. Such theoretical treatments based on the Markov approximation describe only the qubit dynamics in the time-asymptotic quasi-stationary state. Mesoscopically, the qubit decoherence occurs in the time scale of the same order of the tunneling-electron correlation in the QPC, where the non-Markovian dynamics of the qubit is significant [19]. To study the qubit decoherence in the non-Markovian regime, a Lorentzian-type spectral density for the QPC reservoir is considered here

$$J(\epsilon, \epsilon') = \frac{\Gamma_d W^2}{(|\epsilon - \epsilon'| + w)^2 + W^2}, \quad (13)$$

where the constant Γ_d specifies the decay rate of the qubit state due to the interaction of the qubit electron with the QPC, and the factor $|\delta\Omega|^2$ in Eq. (12) has been absorbed into Γ_d . The spectral width W determines the correlation time scales of tunneling electrons across the QPC barrier, and w is an additional parameter to characterize the variation of the QPC internal structure due to the interaction with the qubit electron. In the literature, the wide band limit, $W \gg \gamma$ [so that $J(\epsilon, \epsilon') \rightarrow \Gamma_d$], has been often used to specify the QPC reservoir structure which simply leads to the Markov limit at the time-asymptotic quasi-stationary state [15, 16, 17]. In Fig. 7, the reservoir time correlation function is plotted. The corresponding correlation time scale is determined by the half width of the profile. The wider the half width of $|k(t)|$ is, the longer correlation time scale the tunneling-electron fluctuation maintains. The correlation time scale increases with W decreasing.

To make the qubit decoherence feature apparent due to the QPC measurement, we concentrate again on the charge qubit with symmetric coupled dots $E_1 = E_2$. Thus the energy scale of qubit dynamics is simply characterized by $\gamma = 2\Omega_0$ ($\theta = \pi/2$). The

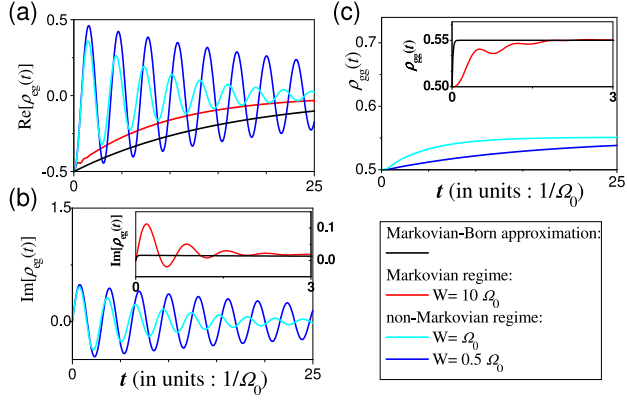


Fig. 8 (color online). The charge qubit dynamics for different width W . The charge qubit is set initially in the $|1\rangle$ state. The other parameters are taken as the same as in Fig. 7. (a) and (b) for the charge qubit dephasing, and (c) the charge qubit relaxation.

master motion for the reduced density matrix becomes,

$$\dot{\rho}_{eg}(t) = -i\gamma\rho_{eg}(t) - 2 \int_{t_0}^t d\tau |k(t-\tau)| \cos[\phi(t-\tau)] \{\rho_{eg}(\tau) - \rho_{ge}(\tau)\}, \quad (14a)$$

$$\dot{\rho}_{gg}(t) = 2 \int_{t_0}^t d\tau |k(t-\tau)| \{\cos[\phi_{\gamma}^{-}(t-\tau)]\rho_{ee}(\tau) - \cos[\phi_{\gamma}^{+}(t-\tau)]\rho_{gg}(\tau)\}, \quad (14b)$$

where the matrix elements are defined as $\rho_{ij}(t) = \langle i|\rho(t)|j\rangle$ with $|i, j\rangle$ being the ground state or the excited state of the qubit, $\phi(t)$ is a time-dependent phase of the reservoir correlation function $k(t-\tau) = |k(t-\tau)|e^{-i\phi(t-\tau)}$, and we have also defined $\phi_{\gamma}^{\pm} \equiv \phi(t-\tau) \pm (t-\tau)\gamma$. The qubit dynamics with different W are simulated in Fig. 8. As we can see, with a very large width $W (= 10\Omega_0)$ the charge qubit undergoes severe decoherence, where the time correlation of tunneling-electron fluctuation is smeared, and the reservoir memory effect on the qubit dynamics is washed out. In other words, the qubit decoherence dynamics is Markov in this regime, which almost coincides with the result of the Born-Markov approximation obtained in the literature [15, 16, 17], see the black curves in Fig. 8. However, if the QPC structure can be recasted such that the width W is comparable to the energy scale of the charge qubit, then non-Markovian processes become dominant in the qubit dynamics. Tunneling electrons in this case propagate with a larger correlation time. The charge qubit simply undergoes an oscillation, see Fig. 8. It is very interesting to see that the qubit decoherence is indeed suppressed in this case. On the other hand, when a large bias voltage is applied, it usually shortens the correlation time of the tunneling-electron fluctuation. A large amount of electrons tunneling across the QPC barrier at a large bias leads to a sharp profile for the time correlation function $|k(t)|$ such that the qubit dynamics evolves into a severe decoherence. However, it is interesting to see from Fig. 8 that even though a large bias voltage ($V_d = 20\Omega_0$) is applied, the qubit decoherence can still be suppressed if the QPC is fabricated to reach a narrower profile of the spectral density. The qubit states can then be read out by a significant current signal without much decoherence. In conclusion, more useful non-Markovian dynamics can be manifested when the explicit

effect of a realistic spectral density with a finite correlation time scale is taken into account, where the qubit decoherence can be suppressed during the QPC measurement.

5 Prospection

In this article we have made an extensive discussion on the non-Markovian decoherence dynamics of a double dot charge qubit induced from the coherence controls and the QPC measurement, using the master equation we obtained from the Feynman-Vernon influence function approach and the Keldysh non-equilibrium Green function technique. For quantum information processing, decoherence dynamics of individual qubit is no doubt a key concern in these electronic systems. On the other hand, decoherence dynamics of entanglement between two qubits [30,31] is another important issue for quantum information processing. Some progress on decoherence entanglement dynamics in cavity optical systems has been reported based on exact master equations [11,12,13]. Further applications to the decoherence dynamics of electron charge and spin entanglement in terms of the quantum dots [32,33] are expected. The full non-equilibrium electron dynamics and the real time monitoring of spin polarization processes in quantum dots are future topics. These together with other physical properties in various nanostructures, such as Kondo effect and Fano resonance, etc., as well as the transient dynamics of electronic quantum transports are under progress. In short, the theory we discussed in this work can be used to study not only the problem of decoherence but also many other interesting physical phenomena in various nanostructures.

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